MATH 315, SPRING 2023 PRACTICE MIDTERM 1

FEBRUARY 21

Each problem is worth 10 points.

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Problem 1. Prove that the column rank and row rank are equal for an $m \times n$ matrix.

Problem 2. Prove that a linearly independent list of vectors in a vector space V may be extended to a basis of V.

Problem 3. Prove that an $n \times n$ matrix A is invertible if and only if its column vectors are linearly independent.

Problem 4. Express
$$\begin{pmatrix} 1 & 2 & 0 & 0 & 0 \\ 2 & 1 & 2 & 0 & 0 \\ 0 & 2 & 1 & 2 & 0 \\ 0 & 0 & 2 & 1 & 2 \\ 0 & 0 & 0 & 2 & 1 \end{pmatrix}$$
 as t

as the product of elementary matrices.

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Problem 5. Let V be finite dimensional, and let $\phi_1, \phi_2, ..., \phi_m$ be linearly independent linear functionals in V'. Prove that

 $\dim((\operatorname{null}\phi_1)\cap\cdots\cap(\operatorname{null}\phi_m))=\dim V-m.$

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