MATH 311, FALL 2024 MIDTERM 1

SEPTEMBER 25

Each problem is worth 10 points.

Problem 1.

a. State the Chinese Remainder Theorem.

b. Find an integer n that satisfies the congruences $n \equiv 1 \bmod 4$, $n \equiv 2 \bmod 5$, $n \equiv 7 \bmod 11$.

Problem 2.

a. State the Euclidean algorithm.

b. Using the Euclidean algorithm or otherwise find $g = \mathrm{GCD}(91, 1001)$ and find integers x, y so that $91x + 1001y = g$.

Problem 3. State Fermat's Little Theorem and give a proof.

Problem 4. Find a primitive root modulo $343 = 7^3$.