MATH 311, FALL 2020 PRACTICE MIDTERM 2

OCTOBER 28

Each problem is worth 10 points.

Problem 1. Define an elliptic curve and give the addition law for points on an elliptic curve. Prove that the addition law is commutative.

Problem 2.

a. Define the Hamiltonians used in the proof of Lagrange's theorem on the sum of four squares.

b. Prove that if q_1 and q_2 are Hamiltonians, the norm of $q_1 q_2$ is the product of the norms.

Problem 3.

a. State the principle of inclusion and exclusion.

b. A permutation $\sigma : \{1, 2, ..., n\} \to \{1, 2, ..., n\}$ is a derangement if $\sigma(j) \neq j$ for all j. Using the principle of inclusion and exclusion or otherwise, calculate the number of permutations of $\{1, 2, ..., n\}$ which are derangements.

Problem 4. Given infinite continued fraction $\langle a_0, a_1, a_2, ...\rangle$ define recursive sequences

$$h_{-2} = 0,\ h_{-1} = 1,\ h_i = a_i h_{i-1} + h_{i-2}$$
$$k_{-2} = 1,\ k_{-1} = 0,\ k_i = a_i k_{i-1} + k_{i-2}.$$

Explain why $r_n = \frac{h_n}{k_n}$ gives the sequence of convergents to the continued fraction and prove that (r_n) converges.