

MATH 311, FALL 2020 PRACTICE MIDTERM 1

SEPTEMBER 23

Each problem is worth 10 points.

Problem 1. Find all pairs of integers (x, y) such that $23x + 81y = 1$ and explain why your list is complete.

Problem 2. Find a reduced quadratic form $ax^2 + bxy + cy^2$, satisfying either $-|a| < b \leq |a| < |c|$ or $0 \leq b \leq |a| = |c|$, which is equivalent to $2x^2 + 5xy + y^2$.

Problem 3.

a. State Hensel's Lemma.

b. Find a solution to $x^2 \equiv 5 \pmod{19^2}$.

Problem 4.

a. State the Pigeonhole Principle.

b. Prove that there are integers a, b with $|a|, |b| \leq 1000$, not both zero, such that $|a + b\sqrt{2}| < \frac{1}{200}$.

