MATH 307, FALL 2020 PRACTICE MIDTERM 2

OCTOBER 26

Each problem is worth 10 points.

Problem 1. Find all critical points of $f(x, y) = x^4 - x^2 + y^2$ and determine if each is a local min, a local max or a saddle point.

Problem 2.

a. Maximize $x^3 + 2y^3$ on $\{x^2 + y^2 \leq 1\}$.

b. Maximize $x^3 + y$ on $\{x^2 + y^2 = 1\}$.

Problem 3. Determine if the function

$$f(x,y) = \begin{cases} \frac{x^3-y^3}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

is continuous and differentiable at 0.

Problem 4. Let $f(x, y, z) = \begin{pmatrix} e^{xyz} \\ xy \end{pmatrix}$, and $g(u, v) = u^2 + v^2$. Calculate f', g' and $(g \circ f)'$.

Problem 5. Let $F(u,v) = \begin{pmatrix} u^3 - v^3 \\ 3u^2 v \end{pmatrix}$. Find $F'(1,2)$ and $(F^{-1})'(-7,6)$.

Problem 6. Find the volume of the largest rectangular solid with sides parallel to the coordinate planes, which fits inside $\frac{x^2}{9} + \frac{y^2}{4} + z^2 = 1$.