MATH 141, FALL 2016, HW7

DUE IN SECTION, OCTOBER 18

Problem 1. Two quickies:

a. Let $\{a_n\}_0^\infty$ and $\{b_n\}_{n=0}^\infty$ be sequences with limits $a_n \to a$ and $b_n \to b$ as $n \to \infty$. If $a_n \leq b_n$ for all n then $a \leq b$.

b. (Integral triangle inequality) Let $f : [a, b] \to \mathbb{R}$ be integrable. Then $|f(x)|$ is integrable, and

$$\left| \int_a^b f(x)dx \right| \leq \int_a^b |f(x)|dx.$$

Problem 2. Construct a differentiable function $f : \mathbb{R} \to \mathbb{R}$ such that $f(x) = 0$ for $x \leq 0$, $f(x) = 1$ for $x \geq 1$ and $0 < f(x) < 1$ for $0 < x < 1$.

Problem 3. Let

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0 & x = 0 \end{cases}.$$

be the function from Lecture which is differentiable, but not continuously differentiable. Let $g(x) = x + 2f(x)$. Show that $g'(0) > 0$ but that g is not increasing in any open interval about 0.

Problem 4. Prove *Bernoulli's inequality*: If $x > -1$ then for all positive integers n,

$$(1 + x)^n \geq 1 + nx.$$

(Hint: prove that the difference has a global extremum at 0.)

Problem 5. Given n real numbers $a_1, ..., a_n$, prove that the sum $\sum_{k=1}^n (x - a_k)^2$ is smallest when x is the arithmetic mean of $a_1, ..., a_n$.

Problem 6. What is the radius of the smallest circular disc required to cover every isoceles triangle of perimeter L?

Bonus Problem. Let $f : [0,1] \to \mathbb{R}$ be a differentiable function. Assume there is no point x in $[0,1]$ such that $f(x) = f'(x) = 0$. Prove that f has finitely many zeros in $[0,1]$. (Hint: use the method of bisection.)