MATH 141, FALL 2016 PRACTICE MIDTERM 2

NOVEMBER 2

Solve 4 of 6 problems. You may quote any result stated during lecture, so long as you represent the result accurately.

Problem 1.

a. (2 points) State carefully the Chain Rule of differential calculus.

b. (3 points) For integer $n \geq 1$, define the n-times iterated logarithm by $\log_{(1)} x = \log x$, and, for $n \geq 1$, and x such that $\log_{(n)}(x) > 0$, $\log_{(n+1)} x = \log(\log_{(n)} x)$. Derive a formula for $\frac{d}{dx} \log_{(n)} x$.

Problem 2.

a. (2 points) State carefully the Mean Value Theorem for a function on an interval $[a, b]$.

b. (3 points) Prove that if f is n-times differentiable on (a, b) and $f(x) = 0$ for $n + 1$ different x in (a, b), then $f^{(n)}(x) = 0$ for some $x \in (a, b)$.

Problem 3. Use integration by parts to derive the formula for $m, n \geq 1$,

$$\int \frac{\sin^{n+1} x}{\cos^{m+1} x} dx = \frac{1}{m} \frac{\sin^n x}{\cos^m x} - \frac{n}{m} \int \frac{\sin^{n-1} x}{\cos^{m-1} x} dx.$$

Apply the formula to integrate $\tan^2 x$ and $\tan^4 x$.

Problem 4. Evaluate

$$\lim_{x\to\infty} xe^{\frac{x^2}{2}} \int_x^\infty e^{-\frac{t^2}{2}}\,dt.$$

Problem 5. Prove the following *Integral Cauchy-Schwarz Inequality.* Let f and g be continuous functions on $[a, b]$. Then

$$\left(\int_a^b f(x)g(x)dx\right)^2 \leq \int_a^b f(x)^2 dx \int_a^b g(x)^2 dx$$

with equality if and only if $f = cg$ or $g = cf$ for some $c \in \mathbb{R}$.

Problem 6. A right triangle with hypotenuse of length a is rotated about one of its legs to generate a right circular cone. Find the greatest possible volume of such a cone.