

MAT 132

Practice Midterm II.

March 28

This is a closed notes/ closed book/ electronics off exam.

Please write legibly and show your work.

Each problem is worth 20 points.

Full Name:						
Problem	1	2	3	4	5	Total
Grade						

Problem 1. Find the volume of a solid whose base is a circle of radius 1, and whose horizontal cross-sections are squares.

Problem 2. Let R be the region

$$R = \{(x, y) : 0 \leq x \leq \pi, 0 \leq y \leq \sin x\}.$$

a. Find the center of mass of R .

b. Let S be the solid found by rotating R about the y axis. Find the volume of S .

c. Let T be the solid found by rotating R about the x axis. Find the volume of T .

Problem 3. Find the length of a curve given by the parametric equations $x(t) = e^t \cos t$, $y(t) = e^t \sin t$, $0 \leq t \leq \pi$. Find the average x and y coordinates over this interval.

Problem 4. A 5kg mass is attached to a spring with spring constant $K = 20kg/s^2$. The spring is stretched to 10 cm and then released, after which it exhibits simple harmonic motion with displacement from equilibrium in cm

$$x(t) = 10 \cos(2t).$$

Find the work done in Joules on the mass by the spring between $t = 0$ and $t = \frac{\pi}{2}$.

Problem 5.

- a. Given the initial value problem $y' = 2(x + y - 1)$, $y(0) = 1$, use Euler's method with step $h = \frac{1}{2}$ to estimate $y(2)$.

- b. Find the equilibria of the time homogeneous differential equation $y' = \cos(\pi y)$. An equilibrium $y = a$ is called stable if there is a small interval $I = [a - \delta, a + \delta]$ such that if $y(0) \in I$ then $y(t) \rightarrow a$ as $t \rightarrow \infty$. Which equilibria are stable?

