MAT 132

Practice Final Exam. May 9, 2018

This is a closed notes/ closed book/ electronics off exam.

Please write legibly and show your work.

Each problem is worth 20 points.

Full Name:				
Problem	1	2	3	4
Grade				
Problem	5	6	7	Total
Grade				

Problem 1. Decide whether each series converges or diverges. Justify your answer.

a. $\sum_{n=1}^{\infty} \frac{(-1)^n}{1+\log n}$

b. $\sum_{n=1}^{\infty} \frac{n+1}{e^{\sqrt{n}}}$

c. $\sum_{n=1}^{\infty} \sin\left(\frac{(-1)^n}{n}\right)$

d. $\sum_{n=1}^{\infty} \frac{n-3}{n^3+1}$

Problem 2.

a. Find the Taylor series of $\sqrt{x}$ about $x = 1$ and determine the interval of convergence.

b. Evaluate as an infinite series $\int e^{-t^2} dt$.

Problem 3. Approximate $\sin\left(\frac{1}{2}\right)$ correct to within 10^{-6} by performing a Taylor expansion about 0.

Problem 4. Solve the following initial value problems.

a. $\frac{dy}{dx} = x(1 - y^2)$, $y(0) = 0$.

b. $\frac{dy}{dx} = yx\sqrt{1 + x^2}$, $y(0) = 1$.

Problem 5.

a. Find the length of the curve (e^{2t}, e^{3t}), $0 \leq t \leq 10$.

b. Find the center of mass of the figure

$$A = \{(x, y) : -5 \leq x \leq 5, x^2 \leq y \leq 25\}.$$

Problem 6. Perform the following indefinite integrals.

a. $\int \frac{x}{1+x^4} dx$

b. $\int x^3 \sin x dx$

c. $\int \frac{dx}{x^3+5x}$

d. $\int \frac{dx}{x(\log x)^2}$.

Problem 7. The population of wolves (W) and rabbits (R) in an ecosystem is governed by the predator-prey equations

$$\frac{dR}{dt} = 0.1R - .002RW, \qquad \frac{dW}{dt} = -0.01W + 0.0002RW.$$

Find any equilibrium points for the system, and derive the family of curves describing the periodic $R - W$ phase trajectories.