MAT 540, Homework 6, due Wednesday, Oct 9

From Lee's textbook: questions **6-9, 6-10, 6-13ab.**

1. (a) Prove Lemma 5.41 (this is exercise 5.42 in the text). Read the discussion in the book surrounding the lemma.

(b) Let M be an orientable smooth manifold with boundary. Prove that its boundary ∂M is orientable.

Note: If M is non-orientable, then ∂M might or might not be orientable. The boundary of the Möbius band is the circle (orientable). On the other hand, the Klein bottle (non-orientable) is a boundary of a (non-orientable) 3-manifold. Try to find this 3-manifold (you don't have to submit this part).

2. In a standard $\mathbb{R}^n$, the translation by a vector $v \in \mathbb{R}^n$ is the map $x \mapsto x + v$.

Let X, Y be two submanifolds in $\mathbb{R}^n$. Show that there exists a translation τ_v by a vector v of length less than a given $\epsilon > 0$, such that $\tau_v(Y)$ is transverse to X. In particular:

(i) If C_1, C_2 are two simple closed curves in $\mathbb{R}^2$, then after an arbitrarily small translation of one of the curves, the two curves will intersect at finitely many points. (A simple closed curve is a compact connected embedded 1-dimensional submanifold without boundary.)

(ii) If C_1, C_2 are two simple closed curves in $\mathbb{R}^3$, then by an an arbitrarily small translation of one of the curves, you can make the two curves disjoint.

(iii) If S_1, S_2 are two embedded closed surfaces in $\mathbb{R}^3$, then after an arbitrarily small translation of one of the surfaces, the two surfaces will intersect along a finite collection of simple closed curves. (These curves may be knotted and linked. A closed surface = compact 2-dimensional manifold without boundary.)

Explain briefly how these corollaries follow. Would you be always able to achieve transversality if you only consider translations by vectors parallel to a given vector v_0? Or by vectors of unit length?