MAT 540, Homework 13, due TUESDAY, DEC 3 (note special date)

1. Let X, X' be two homotopy equivalent topological spaces, $f : X \to X'$ a homotopy equivalence. Let $\phi : \partial D^n \to X$ be continuous. Show that

$$X \sqcup_\phi D^n \sim X' \sqcup_{f\circ\phi} D^n,$$

more precisely, the map $X \sqcup_\phi D^n \to X' \sqcup_{f\circ\phi} D^n$ determined by f and id_{D^n} is a homotopy equivalence.

Note: Together with Question 5 Homework 12, this is another trick that's useful when looking for homotopy equivalences between CW spaces.

2. (a) Let X be the space obtained from a torus $S^1 \times S^1$ by attaching a Möbius band via a homeomorphism from the boundary circle of the Möbius band to the circle $S^1 \times \{x_0\}$ in the torus.

Compute $\pi_1(X)$ and describe the universal covering $p_0 : \tilde{X}_0 \to X$.

Choose a non-trivial subgroup of $\pi_1(X, x_0)$ (your choice), describe the covering $p : \tilde{X} \to X$ that corresponds to your subgroup, and also describe the covering $p_0 : \tilde{X}_0 \to \tilde{X}$ by the universal covering space X_0 that is provided by the hierarchy of coverings. (The composition $p \circ p_0$ should be the universal covering $\tilde{X}_0 \to X$.) If you have trouble seeing the universal covering, it might actually be easier to go in stages, finding some other non-trivial covering first.

Find $Deck\, \tilde{X}_0$ and $Deck\, \tilde{X}$, and describe how the deck transformations act on $\tilde{X}_0$ and on $\tilde{X}$.

(b) Please answer the same questions for for the space Y obtained by attaching a Möbius band to $\mathbb{R}\mathrm{P}^2$ via a homeomorphism from its boundary circle to the circle in $\mathbb{R}\mathrm{P}^2$ formed by the 1-skeleton of the usual CW structure on $\mathbb{R}\mathrm{P}^2$.

Please also do questions **12, 24 from Hatcher section 1.3**. Question 2 above is adapted from question 21 section 1.3 in Hatcher.

I will add one more question about the action of the fundamental group on the fiber.
An update will be posted soon! I'll send an email alert when updated.