MAT 539 Algebraic Topology

Problem Set 4

due Monday, November 22

We don't have a class on Nov 22 (it's a Thursday on University calendar). Please slide the completed assignment under my door (3-107) by the end of the day on Nov 22.

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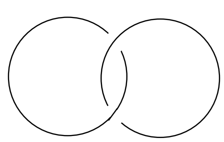
Problem 1. Let $C(X)$ be the singular chain complex for a topological space X. We can define cohomology with coefficients in an abelian group G by dualizing via $\mathrm{Hom}(C, G)$, or, alternatively, we can first consider the singular chain complex $C(X; G)$ with coefficients in G, and then dualize. Check that the result will be the same.

Problem 2. The CW-complex X is obtained from the $2n$-dimensional CW-complex $\mathbb{R}\mathrm{P}^{2n}$ by attaching a single $(2n+1)$-cell. Show that the homology groups of X (with any coefficients) are the same as those of $\mathbb{R}\mathrm{P}^{2n+1}$. What about cohomology groups?

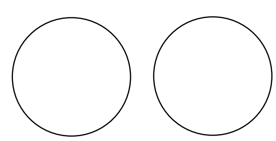
Problem 3. Compute
(a) cohomology groups $H^*(\mathbb{R}\mathrm{P}^7, \mathbb{Z}/6)$ (use the universal coefficients theorem; check your answer via cellular cohomology)
(b) homology groups $H_*(\mathbb{R}\mathrm{P}^3 \times K)$ with $\mathbb{Z}$ coefficients and $\mathbb{Z}/4$ coefficients
(c) cohomology ring of $S^1 \times S^1 \times S^3$, with $\mathbb{Z}$ coefficients
(d) cohomology ring of $T \times K$ with $Z/2$ coefficients
In this question, T is the torus, and K is the Klein bottle.

Problem 4. Let A be the union of two linked circles, B the union of two unlinked circles, as shown below. Prove that their complements $S^3 - A$ and $S^3 - B$ have isomorphic cohomology groups, but not isomorphic cohomology rings.

A



B



Hatcher 9, 11 p.205 (you don't need to know anything about Moore spaces, but you can look them up).
Read example 3.13 on p. 215 of Hatcher. Do Question 1 p. 228.
Also, please read the proof of Thm 3.14 on p. 215.