MAT 539 Algebraic Topology

Problem Set 2

due Wednesday, October 6

Read about split exact sequences in Hatcher (p.147-148). This is often useful for calculations.

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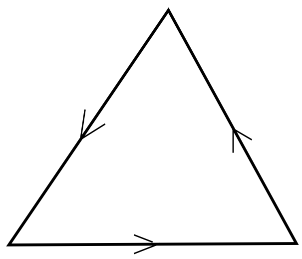
Problem 1. Let X be the topological space obtained by identifying all three sides of a triangle as shown in the diagram.



Is X a manifold? Prove your answer.

(Recall that a topological manifold of dimension n is a Hausdorff second-countable space where every point has a neighborhood homeomorphic to $\mathbb{R}^n$.)

Problem 2. Compute the homology of the 2-torus T via the Mayer-Vietoris sequence applied to $X = U \cup V$, where U and V are two cylinders: if T is obtained by identifying the opposite sides of the square $S = [-1, 1] \times [-1, 1]$ in the (x, y)-plane, take

$$U = \{(x, y) \in S : |x| < 2/3\}, \qquad V = \{(x, y) \in S : |x| > 1/3\}.$$

Also, identify the generators for $H_*(T)$ from your calculation.

(This is probably the least convenient way to compute homology of the torus, but it is an instructive exercise with exact sequences.)

Problem 3. A knot in $\mathbb{R}^3$ (or in S^3) is a smoothly embedded circle. Given a knot K in $\mathbb{R}^3$, let $\nu(K)$ be its tubular neighborhood. The space $\mathbb{R}^3 - \nu(K)$ is called the knot complement. Similarly, one can consider knot complements for knots in S^3.

Compute the homology $H_*(\mathbb{R}^3 - \nu(K))$ of the knot complement for $K \subset \mathbb{R}^3$. Also, compute $H_*(S^3 - \nu(K))$ for $K \subset S^3$. Identify the generators as explicit cycles.

Please also do the following questions from Hatcher: 28 p.133, 1, 7, 8 p. 155.