MAT 530 Topology, Geometry I

Problem Set 8

due Friday, November 6

1. Suppose X is a second-countable space, and $X/\sim$ is the quotient space with the respect to some equivalence relation $\sim$. Is $X/\sim$ necessarily second-countable? Prove or give a counterexample.

(Unlike the exam question, there are no additional hypotheses on X and $\sim$.)

2. The Hatcher's book has an appendix on cell complexes. Please read the definitions (we discussed them in class) and Proposition A.1. Prove (from definition on p.519) that every CW-complex is Hausdorff. You can find a proof in Hatcher, but you should understand it thoroughly and write in your own words.

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Please read the definition of a topological group on p. 145 in Munkres, and do question 2 on pp.145–146.

3. Let G be a topological group, $x_0 \in G$. Show that the fundamental group $\pi_1(G, x_0)$ is abelian.

4. Let X, Y be topological spaces, $x_0 \in X$, $y_0 \in Y$. Show that

$$\pi_1(X \times Y, (x_0, y_0)) = \pi_1(X, x_0) \times \pi_1(Y, y_0).$$

Please also do Questions 2 and 3 of §51.