MAT 530 Topology, Geometry I

Problem Set 3

due Friday, September 25

Problem 1. Consider two topological spaces X and X', defined as follows.
The space X is a subspace of the standard $\mathbb{R}^2$, given in the polar coordinates by

$$X = \{(r, \theta) : 0 \le r \le 1,\ \theta = \frac{2\pi}{n} \text{ for some } n \in \mathbb{N}\}.$$

The space X' is a subspace of $\mathbb{R}^\omega$, where the latter is the space of sequences with the metric topology induced by the metric

$$d((x_1, x_2, \dots x_n, \dots), (y_1, y_2, \dots y_n, \dots)) = \sup_i |x_i - y_i|.$$

(You do not have to check that this is indeed a metric.)
Define $X' = \cup_{n \ge 1} Y_n$, where

$$Y_n = \{(y_1, y_2, \dots, y_n, \dots) : 0 \le y_n \le 1,\ y_i = 0 \text{ for all } i \ne n\}$$

(i.e a sequence is in Y_n if its n-th coordinate is between 0 and 1, and all other coordinates vanish).
Show that X and X' are *not* homeomorphic (despite an obvious bijection between them).

Problem 2. Suppose X is connected, $f : X \to \mathbb{R}$ is locally constant. (This means that every point $x \in X$ has a neighborhood $U \ni x$ such that f is constant on U.) Show that f is then a constant function.

Problem 3. Suppose $U \subset \mathbb{R}^n$ is open and connected. Show that U is path-connected.

Please also do questions 11 of §17, 9, 13 of §18, 4 of §23, and 1, 2 of §24 of Munkres.

Required Reading: please read §25.