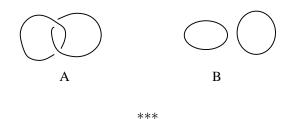
MAT 530 Topology, Geometry I

Problem Set 10 due Friday, November 20

1. Prove that the Möbius band does not retract to its boundary circle.

**2.** Let A be the union of two linked circles in  $\mathbb{R}^3$ , B the union of two unlinked circles, as shown below. Prove that the complements  $\mathbb{R}^3 - A$  and  $\mathbb{R}^3 - B$  have non-isomorphic fundamental groups, thus showing that no homeomorphism  $\mathbb{R}^3 \to \mathbb{R}^3$  takes A to B.



Do Question 9 of section 58 in Munkres to learn about degrees.

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The following questions use the van Kampen theorem. (We will finish the proof in class on Monday and get some practice with applications.)

**3.** For every n > 2 construct a (path-connected) topological space X such that  $\pi_1(X) = \mathbb{Z}/n$ .

4. Let the space X be the quotient space of  $S^2$  obtained by identifying the north and south pole. Compute  $\pi_1(X)$ . (There are several ways to do this.)

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Please also do question 8 from section 1.2 of Hatcher (p.53).

Read in Munkres about the calculation of the fundamental groups of n-dimensional spheres from scratch. (We'll be lazy and just apply the van Kampen theorem in class.)