MAT 530 Topology, Geometry I

## Problem Set 10

due Friday, November 20

**1.** Prove that the Möbius band does not retract to its boundary circle.

**2.** Let $A$ be the union of two linked circles in $\mathbb{R}^3$, $B$ the union of two unlinked circles, as shown below. Prove that the complements $\mathbb{R}^3 - A$ and $\mathbb{R}^3 - B$ have non-isomorphic fundamental groups, thus showing that no homeomorphism $\mathbb{R}^3 \to \mathbb{R}^3$ takes $A$ to $B$.

A B

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Do Question 9 of section 58 in Munkres to learn about degrees.

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The following questions use the van Kampen theorem. (We will finish the proof in class on Monday and get some practice with applications.)

**3.** For every $n > 2$ construct a (path-connected) topological space $X$ such that $\pi_1(X) = \mathbb{Z}/n$.

**4.** Let the space $X$ be the quotient space of $S^2$ obtained by identifying the north and south pole. Compute $\pi_1(X)$. (There are several ways to do this.)

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Please also do question 8 from section 1.2 of Hatcher (p.53).

Read in Munkres about the calculation of the fundamental groups of n-dimensional spheres from scratch. (We'll be lazy and just apply the van Kampen theorem in class.)