MAT 511 Fundamental Concepts of Math

Problem Set 7

due Thursday, Oct 30

Please prove all your answers. Short and elegant proofs are encouraged but not required.

Problem 1. The union of n sets $B_1, B_2, \dots, B_n$ can be defined as

$$B_1 \cup B_2 \cup \dots B_n = \{x : x \in B_i \text{ for some } i,\ 1 \le i \le n\};$$

informally, it's the set of elements from all the sets $B_1, \dots, B_n$, put together.

(a) Show that

$$B_1 \cup B_2 \cup B_3 \cup \dots B_{n-1} \cup B_n = (B_1 \cup B_2) \cup B_3 \cup \dots B_{n-1} \cup B_n = (B_1 \cup B_2 \cup B_3 \cup \dots B_{n-1}) \cup B_n$$

(b) Prove that $A \cap (B_1 \cup B_2 \cup \dots B_n) = (A \cap B_1) \cup (A \cap B_2) \cup \dots (A \cap B_n)$ in two ways: (i) using induction and (ii) not using induction.

Problem 2. There are 70 students in a class. 27 take math, 22 take bio, 32 take chem; 8 of those who take math also take bio, 6 of those who take bio also take chem, 10 of those who take chem also take math. 3 students take math, bio, and chem. How many students take none of these three classes?

Problem 3. 10% of all mathematicians are philosophers, while 15% of all philosophers are mathematicians. Are there more mathematicians or more philosophers?

Please also do questions 10dg, 11e, 15cbd, 17fh from section 2.2.