MAT 511 Fundamental Concepts of Math

Problem Set 12

due Thursday, Dec 11

Please prove all your answers. Short and elegant proofs are encouraged but not required.

Problem 1. Consider the following relation on some collection of sets: $A \sim B$ if there exists a bijection $f : A \to B$. Prove that $\sim$ is an equivalence relation.

Problem 2. Let A, B be sets of points in the plane such that A is a circle of radius 1, and B is a circle of radius 100. Describe a one-to-one correspondence between A and B, thus showing that the two circles have the same cardinality ("number of points").

Problem 3. Prove that the following sets are countable by describing an enumeration for each set.

(a) the set of rational numbers in the interval $(1, 2)$

(b) the set of integers that have remainder 2 when divided by 5

(c) $\{(x, y) : x \text{ is a natural number}, y \text{ is a real number}, xy = 1\}$

(d) $A \times B$, where A is finite, B is countable

Please also do questions 13a, 16cd from section 5.3.