

MAT 511 Fundamental Concepts of Math

Problem Set 11

due Thursday, Dec 4

Please prove all your answers. Short and elegant proofs are encouraged but not required.

Please do questions 3bcd, 9abd, 10bd (see below for the explanation of projection functions), 14ab (go back to section 3.2 if you don't remember what \mathbb{Z}_4 , etc stands for), 17bf from section 4.3, and 9 and 17 from section 4.4.

In question 10 from 4.3, projection functions $\pi_1 : A \times B \rightarrow A$ and $\pi_2 : A \times B \rightarrow B$ are defined by $\pi_1(a, b) = a$ and $\pi_2(a, b) = b$, for $(a, b) \in A \times B$.

I have graded the quiz and am very disappointed by the results. I include the quiz questions below, please think about them again (but do not submit the answers). Keep in mind that the function f might not have an inverse; $f^{-1}(b)$, etc stands for the inverse image, which might be an empty set or a set with several elements.

Quiz

Let $f : A \rightarrow B$ be a function, $a \in A$, $b \in B$. Determine whether the following statements are true or false. Prove or give counterexamples.

1. $f^{-1}(f(a)) = \{a\}$
2. $f(f^{-1}(b)) = \{b\}$

If you decide that a statement is false in general, determine whether it is true for all injective or for all surjective functions. (Give a proof.) For example, you can show that a statement is false in general (and give a counterexample) but holds true for all injective functions (and give a proof).