MAT 511 Fundamental Concepts of Math

Problem Set 10
due Thursday, Nov 20

Please prove all your answers. Short and elegant proofs are encouraged but not required.

Problem 1. Suppose X is a set with an equivalence relation $\sim$, $X/\sim$ is the set of equivalence classes.
Let the function $f : X \to X/\sim$ be defined by mapping each element $x \in X$ to its class.
(a) Prove that f is always surjective.
(b) Suppose f is injective. Describe the equivalence relation $\sim$.

Problem 2. Let $f : X \to Y$ be a surjective function, $A, B \subseteq Y$.
Suppose $f^{-1}(A) \subseteq f^{-1}(B)$. Does it follow that $A \subseteq B$? Prove or give a counterexample.

Please also do questions 4,6, 8bcd from 4.3, and 8, 14be, 18 from 4.4. In 18, also determine whether the statement remains true if the function is not bijective.