MAT 364, Homework 11, due Thursday, Dec 1, in class

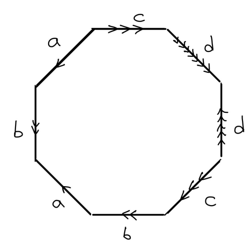
1. In Topoland, there are 100 towns and 102 roads, each road connecting 2 towns. To travel from one town to another, one can follow several roads one after the other (connecting at the towns in between); it is possible to get from each town to every other town this way.

The Highway Administration wants to close several roads to reduce expenses. (They choose the roads to close after a careful study.) After the road closure, it should still be possible to get from each town to every other town via open roads.

What is the largest number of roads can be closed? Prove your answer. (Your proof should work for every possible configurations of roads rather than for a specific example.)

We stated an important classification theorem in class: every compact oriented surface (without boundary) is a sphere, a connected sum of tori, or a connected sum of projective planes. Different surfaces on the list are distinguished by the Euler characteristic + whether they are orientable or not. Use this fact in Question 2(d).

2. In the diagram below, the sides of the octagon are glued in pairs, identifying every pair of sides marked by the same letter and so that the directions of the arrows are matched (this may or may not require a twist).



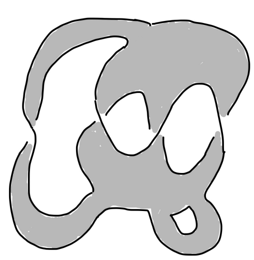
(a) Explain why the resulting quotient space is a surface; indicate briefly why it's compact and connected.

(b) Determine whether this surface is orientable or not.

(c) Compute the Euler characteristic of this surface. You can do this directly from the diagram, but be careful since some edges and vertices are identified after gluing!

(d) Identify this surface as one of the "standard" surfaces on the classification list: a sphere, a connected sum of tori, or a connected sum of projective planes; determine how many tori or projective planes are in the connected sum.

A similar classification theorem holds for surfaces with boundary: every compact oriented surface with boundary is a sphere, a connected sum of tori, or a connected sum of projective planes, with a number of "holes" (disks cut out of the surface). Different surfaces on the list are distinguished by the Euler characteristic, the number of holes, and whether they are orientable or not. Use this fact in Question 3; it's best to do this question after we discuss this material in class (after Thanksgiving).



3. Identify the surface shown on the figure as one of the surfaces as above with a number of holes. (Determine which "standard" surface and how many holes, and prove your answer.) You will have to go through a procedure similar to Question 2.

Please also do questions **14.9, 14.10** from the textbook.