MAT 364, Homework 1 due 9/1

1. Perform the following topological experiment with a Möbius band and describe the results. Recall that a Möbius band can be made from a long rectangular strip of paper: just glue the ends of the strip after a half-twist. We played with these in the first lecture; we learned that the Mobius band has only one side (unlike the case of a usual annular strip with no twist, you cannot color one side red and the other blue: if you start coloring a side and move along, you end up coloring all of the twisted band).

Try to cut the Möbius band in half along its length: what happens? We saw that unlike an untwisted annular band, the Möbius band does not separate after cutting.

Take the new twisted band that you got after cutting up your Möbius band. Repeat the experiment: try to color one side, cut it in half lengthwise. Do you think this would be another Möbius band (just longer and thinner), or is this something different? Explain.

2. Consider the collection $\mathcal{T}$ which consists of $\emptyset$, $\mathbb{R}$, and all finite subsets of $\mathbb{R}$. Check whether the collection $\mathcal{T}$ gives a topology on $\mathbb{R}$.

3. Let X be a set, $p \in X$. Consider the collection $\mathcal{T}$ which consists of $\emptyset$ and all subsets of X that contain p. Show that this collection is a topology on X. (It is called the particular point topology.)

4. Find three topologies $\mathcal{T}_1$, $\mathcal{T}_2$, $\mathcal{T}_3$ on the five-point set $X = a, b, c, d, e$ such that $\mathcal{T}_1$ is coarser than $\mathcal{T}_2$ and $\mathcal{T}_2$ is coarser than $\mathcal{T}_3$, without using either the trivial or the discrete topology. Find another topology $\mathcal{T}$ on X that is not comparable to each of the first three that you found. Explain your answers. Don't forget to make sure that $\mathcal{T}_1$, $\mathcal{T}_2$, $\mathcal{T}_3$, and $\mathcal{T}$ are actually topologies! (Brief explanations suffice.)

5. Let $\mathcal{T}_0$ be the standard topology on $\mathbb{R}$, and $\mathcal{T}_1$ be the topology of finite complements. Are these two topologies comparable? Which one is finer/coarser? Prove your answer. Does one of the topologies have an open set which is not open in the other topology? Explain.

6. (a) Let $\mathcal{T}_1$ be the collection of subsets of $\mathbb{R}$ that consists of $\emptyset$, $\mathbb{R}$, and all open rays $(a, +\infty)$ for $a \in \mathbb{R}$.

(b) Let $\mathcal{T}_2$ be the collection of subsets of $\mathbb{R}$ that consists of $\emptyset$, $\mathbb{R}$, and all closed rays $[a, +\infty)$ for $a \in \mathbb{R}$.

Check whether $\mathcal{T}_1$ and $\mathcal{T}_2$ are topologies on $\mathbb{R}$. **Caution:** be very careful with *infinite* unions.