MAT 364 Topology

Problem Set 9

Solutions

4.3. Answer: if you try to cut the Möbius band into $2k$ parts, the result is k cylinders (each of them will be twisted if you actually do the cutting in practice). To see this, study the planar diagram as in Fig 4.6, or argue that after the first cut in the middle, you will get a cylinder, and the remaining cuts will just cut it into k parts. If you try to cut into $(2k+1)$ parts, the result in one Möbius band (from the middle of the planar diagram) and k cylinders. Use the planar diagram for the proof.

1. There is one boundary component.

2. (a), (d), and (f) are not surfaces – (a) has a "bad" point where the two segments come out of the torus, (d) has a "bad" point at the common vertex of the two deleted triangles, and (f) has all the points inside the cube where the neighborhoods are 3-dimensional (rather than 2-dim disks). The rest are surfaces: (b), (e), (g) surfaces without boundary, (c) a surface with boundary. To check that (e) is a surface, you need to see that the edges are glued up nicely (every point on an edge has a disk neighborhood which is glued up out of two half-disks, and the 4 vertices are glued together, so that the resulting point has a neighborhood glued from 4 quater-disks). To check that (g) is a surface, you can (informally) argue that it becomes a sphere if one blows a lot of air into it. More formally, you can worry about the edges and the corners – check that these have nice neighborhoods. For a corner, a typical neighborhood (intersection with a ball in $\mathbb{R}^3$) consists of three pieces of adjacent faces. Each of them is a quater-disk; after a homeomorphism, we can think of it as one-third of a disk, and then the three glue together to give a disk neighborhood.

3. The connected sum of an arbitrary surface with a disk is the same surface with a hole. Indeed, to form the connected sum, we cut out a hole out of the surface and a hole out of the disk, and glue the boundary circles together. Now, think of the disk with a hole as a cylinder, or better yet, turn it inside out – then the outer circle will be glued to the boundary of the surface with a hole, and the hole in the disk will form a new hole in the surface.