MAT 364 Topology

Problem Set 7

due Wednesday, October 27

Problem 1. Let X be a topological space. Suppose that Y is a subset of X. We can define a topology on Y as follows. If $\mathcal{T}_X = \{U_i\}$ is the topology on X, let $\mathcal{T}_Y$ to be the collection of all sets $U_i \cap Y$. (Notice that $U_i \cap Y$ are subsets of Y.)
Check that $\mathcal{T}_Y$ is indeed a topology on Y (ie it satisfies the axioms). It is called a *subspace topology.*

Problem 2. Suppose that X is a topological space, Y is a subset of X. As explained in Problem 1, Y can be considered as a topological space (equipped with subspace topology). Prove that if X is compact, and Y is closed in X, then Y is also compact.

Problem 3. Consider the set $\mathbb{R}^2$. For any two points $\mathbf{x} = (x_1, x_2)$ and $\mathbf{y} = (y_1, y_2)$, define the distance $d(\mathbf{x}, \mathbf{y})$ by the formula

$$d(\mathbf{x}, \mathbf{y}) = |x_1 - y_1| + |x_2 - y_2|.$$

(a) Prove that d satisfies the axioms for a distance, ie $(\mathbb{R}^2, d)$ is a metric space.

(b) Sketch the unit disk $D(0, 1)$ centered at 0 for this metric.

Please also do Exercise 3.3 p. 40, Exercise 3.9 p. 43.