

MAT 364 Topology

Problem Set 5

due Wednesday, October 6

Problem 1. Determine whether the following sets are connected or not. Prove your answers.

- (a) $X =$ the union of the x -axis and the y -axis in the (x, y) -plane \mathbb{R}^2
- (b) $Y =$ the union of the z -axis and the circle $\{x^2 + y^2 = 1, z = 0\}$ in the 3-space \mathbb{R}^3 (with coordinates (x, y, z))
- (c) $Z = \{x^2 + y^2 + z^2 = 1\}$, ie a sphere in \mathbb{R}^3 .

Problem 2. Suppose X is connected.

- (a) Does $IntX$ have to be connected?
- (b) Does CLX have to be connected?

Prove or give counterexamples.

Problem 3. (a) Show that \mathbb{R} is homeomorphic to $(0, +\infty)$.

(b) Show that \mathbb{R} is *not* homeomorphic to $[0, +\infty)$.

Problem 4. Prove that a circle is not homeomorphic (a) to an open interval, (b) to a closed interval.