MAT 364 Topology

# Exam I checklist

Below is the list of topics, definitions and theorems that we discussed so far. For the exam, you are required to know all the definitions. In your solutions of exam problems, you can refer to any of the theorems and facts we proved in class. You will not be asked to reproduce any of those proofs, nut familiarity with the techniques is expected and will be useful.

## Open and Closed Sets

Interior, exterior, frontier points for subsets of $\mathbb{R}^n$
Open and closed sets in $\mathbb{R}^n$
Closure and interior of a set
Thm: $ClA$ is always closed
A set is open iff its complement is closed
Finite intersections and arbitrary unions of open sets are open
Relatively open/relatively closed sets
Thm: $A \subset X$ is relatively open in $X$ iff $A = O \cap X$ for some $O$ open in $\mathbb{R}^n$

## Continuity

Definition in terms of open sets
$\epsilon$-$\delta$ definition
Equivalence of the two definitions
"In-between" definition: $f^{-1}(N_\epsilon(x))$ is open for any neighborhood $N_\epsilon(x)$
Examples: checking whether a given function is continuous using each of the definitions
Thm: if the sequence $\{x_n\}$ converges to $a$, $f$ is continuous, then $f(x_n) \to f(a)$.
Homeomorphisms, examples (an open disk $D^2$ is homeomorphic to $\mathbb{R}^2$, etc)

## Connectedness

Def: $X$ is connected if it cannot be represented as $A \cup B$, with $A$ and $B$ disjoint non-empty open sets
Thm: $[a, b]$ is connected
Characterization of all connected sets in $\mathbb{R}$
Corollary: intermediate value thm
Application: $[a, b]$ and $(c, d)$ are not homeomorphic
Thm: if a number of connected sets all share a point, the union of these sets is also connected
Application: disk and circle are connected, letter $X$ is connected, etc

## Compactness

Definition of compactness via convergent subsequences
Compactness of $[a, b]$, of $n$-dimensional closed cube
$X$ compact in $\mathbb{R}^n$ iff X closed and bounded

Thm 1: if $Y$ compact, $X$ a subset of $Y$, then $X$ is compact
Thm 2: if $X$ is compact, $f : X \to Z$ continuous, then $f(X)$ compact
Corollary: a continuous real-valued function on a compact set is bounded and attains its maximum
Definition of compactness via open covers and finite subcovers
Proofs of thm 1 and thm 2 via this definition, examples
Equivalence of the two definitions for compact sets in $\mathbb{R}^n$