MAT 360 Geometry

Homework 7

due Thursday, Apr 4

Problem 1. Prove that any isometry maps a circle into a circle of the same radius. (An isometry is a distance-preserving transformation of any kind. Please don't just assume that it has to be a rotation, a reflection, etc. You have to show that if you apply an arbitary isometry to a given circle, the image will always be a circle, as opposed to some weird shape.)

Problem 2. In the xy-plane, let R_x be the reflection about the x-axis, R_y the reflection about the y-axis, R_o the rotation about the origin by $90°$ counterclockwise.

(a) Describe $R_x \circ R_y$ and $R_y \circ R_x$. Are they the same?

(b) Describe $R_x \circ R_o$ and $R_o \circ R_x$. Are they the same?

All of the above compositions will be reflections, rotations or translations. "Describe" means determine which type they are, and give the axis for reflections, the center and angle for rotations, or the vector for translations, as appropriate.

Problem 3. Consider the composition of a reflection R about the line l with the translation T by a vector $\vec{AB}$ parallel to l.

(a) Show that the order in which the transaltion and the reflection are performed does not matter here, i.e. $R \circ T = T \circ R$. Explain why the resulting transformation is an isometry.

(b) Show that $R \circ T$ **cannot** be respresented as a rotation, reflection, or translation.

Problem 4. Prove that the composition of any two translations is a translation. What is the vector for the composite translation?

Problem 5. (a) Prove that the symmetry is a point O (see lecture notes for the definition!) is the same as the rotation about O by $180°$.

(b) Prove that the composition of any two symmetries in a point is a translation. More precisely, $S_B \circ S_A = T_{2\overrightarrow{AB}}$, where S_X denotes the symmetry about point X, and T is the translation by the indicated vector.

Problem 6. How many axes of symmetry can a quadrilateral have? Consider all cases, prove your answer.