MAT 360 Geometry

# Homework 9

due Thursday, Apr 18

**Problem 1.** Prove that the midpoints of the sides of a quadrilateral are always vertices of a parallelogram. Determine under what conditions this parallelogram is (a) a rectangle, (b) a rhombus. **Hint:** use material from sections 93-97.

**Problem 2.** We proved that any isometry of the plane belongs to one of the following types: rotation, translation, reflection or glide reflection.

(a) Which of these types can be obtained as a composition of 2013 reflections?

(b) Which of these types can be obtained as a composition of 2013 rotations?

Give examples for the types that can be obtained, and prove that the remaining types cannot.

**Problem 3.** Prove that the composition of two similarity mappings (with coefficients $k$ resp. $l$) is a similarity mapping. What is its coefficient?

A useful special case is a composition of a similarity transformation and an isometry (the result is a similarity transformation).

**Problem 4.** (a) Prove that any two circles are similar.

(b) Suppose that triangles $\triangle ABC$ and $\triangle A'B'C'$ are such that

$$\angle A = \angle A', \qquad \frac{A'B'}{AB} = \frac{A'C'}{AC}.$$

Prove that these triangles are similar.

Recall our definition of similar figures: you have to show that there exists a similarity transformation that sends one of them to the other. We have checked that homotheties are similarity transformations, and so are isometries.

**Problem 5.** (a) Show that two homotheties with different centers $A, B$ and the same ratio $k$ differ by rotation or translation, i.e. $H_{A,k} = R \circ H_{B,k}$, where $R$ is some rotation or translation that depends on the homotheties.

(b) More precisely, prove that $H_{A,k} = T \circ H_{B,k}$, where $T$ is the translation by the vector $(k-1)\vec{AB}$.

Part (a) follows from part (b), but is much easier to prove.

Please also do questions **332**, **338(a)** from the textbook.