MAT 360 Topology

Problem Set 11, due Thursday, May 12

Problem 1. Prove that

(a) For any two points A, B that are *not* symmetric about a line l, there exists a circle that passes through A and B and is *not* orthogonal to c.

(b) If the point B is *not* the image of the point A under the inversion about a circle c, then there exists a circle that passes through A and B and is *not* orthogonal to c.

In other words, if all circles passing through A and B are orthogonal to the line l (resp. the circle c), then the points A and B are symmetric about this line (resp. this circle).

Problem 2. Prove that a composition of two inversions with the same center is a homothety centered at the same point. Find the coefficient of this homothety in terms of the degrees of the inversions.

Problem 3. In (x, y) plane, let C be a circle of radius 1 centered at the origin, and consider the inversion I about this circle. Find the images under the inversion I of

(a) the line $\{x = 2\}$

(b) the circle c_1 of radius $\frac{1}{2}$, centered at the point $(-\frac{1}{2}, 0)$

(c) the circle c_2 of radius 1, centered at $(3, 0)$.

Justify your answers.

Problem 4. (a) Consider an inversion I centered at O, a line l passing through O, and a circle c passing through O. Show that the angle between the images of l and c under I is the same as the angle between l and c.

(b) Show that if c_1 and c_2 are two circles passing through O, the angle between them is the same as the angle between their images.

Problem 5. (This question will be used in the next one.)
Given the segments AB, CD, construct a segment EF such that $|EF|^2 = |AB| \cdot |CD|$.

Hint: use similarity as in Question 4(a,b) in Homework 10.

Problem 6. Given a line l and two points A, B not on the line, construct a circle C that passes through A and B and is tangent to l.

Hint: Draw a line m through A and B. The case where $m||l$ is relatively easy. If the lines are not parallel, consider the point $M = m \cap l$. Use the degree of this point with respect to the required circle to locate the point T of tangency, and Question 5 to construct T. Then construct (how?) a circle through A, B, and T.

(The two construction problems above are to be solved, as usual, with a compass and straightedge. Please justify your constrcutions.)