MAT 360 Geometry

Problem Set 12, due Thursday, Dec 8

Problem 1. Given a circle (with a marked center O) and a point A, construct the image of A under the inversion about the circle. Use compass and straightedge only. **Hint:** see picture on p.1 of lecture notes. You have to explain how to construct elements of this picture with compass and straightedge, and prove why it produces points related by inversion. Consider two cases:

(a) A inside the circle; (b) A outside of the circle

Problem 2. In class, we showed that under inversion with center O and radius R, a circle c not passing through O goes to a circle c'. The circle c' can be obtained from c by a homothety with center O and coefficient R^2/d^2, where d is the degree of the point O with respect to c. In class, we considered a picture where c does not intersect the circle of inversion.

Examine the case where c intersects the circle of inversion. Show that the same proof works in this case as well. Make accurate pictures showing c and c' when $R > d$, $R < d$, and $R = d$. When $R = d$, what is the angle between c and c'? Justify your answers and pictures.

Problem 3. Prove that a composition of two inversions with the same center is a homothety centered at the same point. Find the coefficient of this homothety in terms of the degrees of the inversions.

Problem 4. In (x, y) plane, let C be a circle of radius 1 centered at the origin, and consider the inversion I about this circle. Find the images under the inversion I of

(a) the line $\{x = 2\}$

(b) the circle c_1 of radius $\frac{1}{2}$, centered at the point $(-\frac{1}{2}, 0)$

(c) the circle c_2 of radius 1, centered at $(3, 0)$.

Justify your answers.

Problem 5. Consider an inversion I centered at O. Show that it preserves angles between

(a) a circle c_1 passing through O and a circle c_2 not passing through O.

(b) a line l not passing through O and a circle c passing through O.

In other words, show that the angles between these lines/circles are the same as the angles between their images at the corresponding intersection points.

Hint for (b): Draw a line m passing though O and decompose the angle between l and c into sum/difference of the angle between l and m and the angle between m and c. For both parts (a) and (b), use the cases where we've already established in class that inversion preserves angles. (See lecture notes.)