REFERENCE SHEET

The sheet gives formulas only and does not specify important properties of the functions and the contours that are required for the formulas to hold.

Trigonometric Functions:

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}, \quad \cos z = \frac{e^{iz} + e^{-iz}}{2}$$

Cauchy Integral Formula and Its Extension:

$$f^{(n)}(z_0) = \frac{n!}{2\pi i} \int_C \frac{f(z)}{(z - z_0)^{n+1}}\, dz \quad (n = 0, 1, 2, \ldots),$$
$$f^{(0)}(z_0) = f(z_0), \quad 0! = 1.$$

Taylor Series:

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n, \qquad a_n = \frac{f^{(n)}(z_0)}{n!}.$$

Examples of power series expansions:

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n, \quad |z| < 1, \qquad e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!}, \quad |z| < \infty,$$
$$\sin z = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{(2n+1)!} \quad |z| < \infty, \qquad \cos z = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n}}{(2n)!} \quad |z| < \infty.$$

Laurent Series:

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n + \sum_{n=1}^{\infty} \frac{b_n}{(z - z_0)^n}, \qquad R_1 < |z - z_0| < R_2,$$
$$a_n = \frac{1}{2\pi i} \int_C \frac{f(z)}{(z - z_0)^{n+1}}\, dz, \quad b_n = \frac{1}{2\pi i} \int_C \frac{f(z)}{(z - z_0)^{-n+1}}\, dz.$$

Calculating Residues:

$$f(z) = \frac{\phi(z)}{(z - z_0)^m},\ m \geq 1, \phi \text{ analytic},\ \phi(z_0) \neq 0,\ \text{then } \operatorname{Res}_{z=z_0} f(z) = \begin{cases} \phi(z_0), & m = 1, \\ \frac{\phi^{(m-1)}(z_0)}{(m-1)!}, & m \geq 2. \end{cases}$$

$$\operatorname{Res}_{z=z_0} \frac{p(z)}{q(z)} = \frac{p(z_0)}{q'(z_0)} \quad \text{if } p, q \text{ analytic near } z_0, p(z_0) \neq 0, q(z_0) = 0, q'(z_0) \neq 0.$$

Cauchy-Riemann Equations:

$$f(z) = u(x, y) + iv(x, y),$$

$f'(z)$ exists at a point $z_0 = x_0 + iy_0 \Leftrightarrow u_x = v_y, u_y = -v_x$

$$f'(z_0) = u_x + iv_x.$$