MAT 342, Homework 9 due 11/6

Questions from the textbook Sec. 65 p. 196-197: questions **2, 8a, 9, 11**.

[The textbook uses the term "Maclaurin series". This simply means "Taylor series centered at 0", that is, the case $z_0 = 0$. We will just say "Taylor series" for everything.]

More questions:

1. Find the Taylor series for $\sin z$ centered at $z_0 = \pi/2$ in two ways:

(a) by computing derivatives;

(b) by using the relation $\sin z = \cos(\pi/2 - z)$ and a known series for $\cos z$.

For what z does your series converge?

2. Find the Taylor series decomposition for the function $f(z) = \dfrac{1}{z+i}$ about the point $z = 2$. Again, do this in two ways:

(a) by computing derivatives

(b) by using algebra to manipulate the expression to a form where it has a factor like $\dfrac{1}{1 - A(z-2)}$ for some A (and some other constant factors, so that the expression equals $\dfrac{1}{z+i}$, and then using geometric series.

(c) In what disk does your series converge? Specify the center and the radius of the disk, justify your answer. Explain how this radius is related to the domain where the function is analytic.

Optional but recommended if you are familiar with rigorous definitions of convergence of series from real analysis: Read sections 60-61 and do some of the exercises on p.185. Everything works in a very similar way to real analysis.