MAT 342, Homework 5 due 10/2

Questions from the textbook Sec. 46 p.132–133: questions **1, 4** (use the parametric representation of the contours to compute the integral directly); Sec. 47 p.138 questions **1a, 2** (use the theorem in section 47, to be discussed in class on Monday).

More questions:

1. Find antiderivatives and computed the following integrals over paths (contours) that connect the given limits of integration. In (b), assume that the path lies entirely in the right half-plane (1st and 4th quadrants). Explain why the answer depends on the endpoints of the path only.

$$(a)\int_0^{\pi+i}\sin(2z)\,dz \qquad\qquad (b)\int_0^{i}\frac{dz}{z+1}$$

2. (a) Let C be the circle of radius R centered at a, with the parameterization $z = a + Re^{it}$, $-\pi \le t \le \pi$. For all integer n, compute

$$\int_C (z-a)^n\,dz.$$

The case $n = -1$ will look different than $n \neq -1$.

(b) For a circle S of radius 2 centered at 0, compute

$$\int_S (z-3)^n\,dz.$$

Comment on why the answers in (b) are similar to or different from (a).

3. Compute the integral

$$\int_C z^i\,dz$$

in two different situations:

(a) Let C be the semicircle from $-i$ to i going through the 4th and 1st quadrants. Take the principal branch

$$z^i = e^{i\,\mathrm{Log}\,z}.$$

(b) Let C be the semicircle from $-i$ to i going through the 3rd and 2nd quadrants. Take the branch

$$z^i = e^{i\log z} \text{ corresponding to } \log z = \ln r + i\theta, \qquad 0 < \theta < 2\pi, \qquad \text{for } z = re^{i\theta}.$$

Why cannot you take the principal branch in this case?