MAT 319 Foundations of Analysis

Review Sheet

The final exam is cumulative and covers everything we learned during the semester. An outline of the material is below. You must know all the definitions and statements of the important theorems and understand all proofs (although you will not be asked to reproduce a proof of a theorem presented in class). You can use (with clear references) all theorems and facts covered in class, but you have to explain all the steps in your proofs.

No books, calculators, or notes will be permitted.

Properties of $\mathbb{N}$ and $\mathbb{R}$

- Properties of sets and functions; images, preimages, injectivity, etc.
- Induction
- Well-Ordering Property of $\mathbb{N}$
- Countable and uncountable sets
- sup, inf, Completeness Property of $\mathbb{R}$
- the Archimedean Property
- Nested Intervals Property

Sequences

- convergence of a sequence (you must be able to use the definition to give proofs)
- convergent sequences are bounded
- alg. operations $+, -, \times, \div$ on convergent sequences respect limits
- the Squeeze Theorem
- subsequences
- a bounded sequence has a convergent subsequence (Bolzano-Weierstrass Theorem)
- (x_n) converges to L iff every subsequence of (x_n) converges to L

Limits of Functions and Continuity

- ϵ-δ definition of the limit of a function at a point; continuity
- examples of functions that have/don't have a limit at a particular point; basic properties (you must be able to use the definition to give proofs)
- A function that has a limit at c is bounded in a neighborhood of c
- algebraic operations on functions respect limits/continuity
- the Squeeze Theorem
- sequential criterion for the existence of limit and for continuity
- limits at infinity, infinite limits
- a continuous function on $[a, b]$ is bounded (Boundedness Theorem)
- a continuous function on $[a, b]$ attains its maximum and minimum (Maximum-Minimum Theorem)
- Location of roots theorem, bisection method; continuous functions have the intermediate value property