MAT 319: REVIEW SHEET FOR MIDTERM 2

The midterm will cover all material studied since last midterm — that is, sequences and their limits, sections 3.1–3.4 and 3.6 of the textbook. There will be no questions which target the material of the first two chapters — but you might still need them: for example, in order to prove results about the limit of monotone sequence, you need to understand what the least upper bound is.

As before, I expect both computing skills and ability to state important results and construct simple proofs. In particular, you must be able to state

- Definition of limit of a sequence (both finite and infinite limits).
- Limit theorems for sequences
- Monotone Convergence Theorem, sequence, Bolzano—Weierstrass theorem.

Also, you must be able to use various criteria for convergence and divergence.

There will be seven short questions on the exam, each worth 10 points. **The best way to prepare for the exam is to go over all the homeworks**. On the next page, there are some additional practice problems.

Here are some sample problems for the exam. Some of the questions are too long for the actual exam, and would be shortened.

1. Are the following sequences convergent, properly divergent, neither? If they are convergent, what are their limits? Prove your claims.

(a) $x_n = \dfrac{(-1)^n n}{n+3}$ (b) $x_n = \dfrac{(-1)^n n}{n^2+3}$ (c) $x_n = \dfrac{n^2}{n+3}$

2. Suppose that (x_n) is a convergent sequence, $\lim x_n \neq 0$. Show that $x_n \neq 0$ for all sufficiently large n. (Use the definition of limit).

3. (a) State the convergence theorem for monotone sequences.
(b) Prove that if a sequence (a_n) is monotone increasing, and has a convergent subsequence (a_{n_i}), then (a_n) is convergent.

4. Let a sequence a_n be defined by $a_1 = 3$, $a_{n+1} = 1 + \sqrt{a_n - 1}$. Prove that $a_n \geq 2$ for all n, and that a_n is convergent. Find the limit.

5. Suppose that (x_n) tends to $+\infty$.
(a) Show that $(x_n - 10)$ tends to $+\infty$.
(b) Show that $(x_n/10)$ tends to $+\infty$.
(c) Is it true that $(x_n - n)$ tends to $+\infty$? Prove your answer.

6. Let (x_n) be a bounded sequence, and let $s = \sup\{x_n : n \in \mathbb{N}\}$. Also, suppose that $s \neq x_n$ for any n. Show that there is a subsequence of (x_n) which converges to s. What if $x_n = s$ for some n?

7. Does there exist a sequence (x_n) whose every subsequence has a subsequence converging to 1, and another converging to -1? Why or why not? Give an example, or prove that such a sequence does not exist.