MAT 319/320: MIDTERM 1

FRIDAY, OCT. 6

Your name:____
(please print)

No books, notes, or calculators. Unless a problem explicilty states "no explanation required", please try to write as detailed an explanation as possible. Explanations should be such that someone who does not know how to solve this problem (but knows all previous material) can follow your arguments and understand what you are doing; if in doubt, ask. **Answers without explanations will get very little partial credit!**

Notation:
$\mathbb{Z}$ — integer numbers
$\mathbb{N}$ — positive integers
$\mathbb{R}$ — real numbers

There are 5 problems in this exam. Each problem is worth 10 pts. You have 50 minutes. Good luck!

	1	2	3	4	5	**Total**
Grade						

1. For each question, circle ALL correct answers. No explanations or justifications required.

(a) Let a, b, and c be real numbers with $|a| = |b| = 2$, $|c| = 5$. Then
(i) $|a + b + c| \leq 9$.
(ii) $|c - a| \leq 3$.
(iii) $|a + b + c| \geq 1$.
(iv) $|a - b| = 0$.

(b) Which of the following sets are denumerable (=countably infinite)?
(i) The set of all real numbers z such that $z \in [3.1, 3.12]$.
(ii) The set of all irrational numbers z such that $z \in [3.1, 3.12]$.
(iii) $\mathbb{N} \times \mathbb{Z}$.
(iv) Set of all integer multiples of 5.

(c) Let S be a non-empty subset of $\mathbb{R}$. Then
(i) S always has an upper bound.
(ii) S has an upper bound if and only if it has a lower bound.
(iii) S has a supremum whenever is has an upper bound.
(iv) S may have two different upper bounds.

2. Let $f\colon A \to B$ be a function.

(a) Show that for any $C, D \subseteq B$, $f^{-1}(C) \cap f^{-1}(D) = f^{-1}(C \cap D)$.

(b) Suppose that $C, D \subseteq B$ are such that $f^{-1}(C) \cap f^{-1}(D) = \varnothing$. Does this imply that $C \cap D = \varnothing$? (if yes, give a proof; if no, give a counterexample.)

3. Prove, using induction, that for any $n \in \mathbb{N}$

$$\frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)}{2 \cdot 4 \cdot 6 \cdot \dots \cdot 2n} \leq \frac{1}{\sqrt{2n+1}}$$

4. (a) State the Archimedean property of real numbers.

(b) Recall that for a real number a, we denote $V_\varepsilon(a) = \{x \mid |a - x| < \varepsilon\}$. Find

$$\bigcap_{n\in\mathbb{N}} V_{1/n}(a)$$

5. Consider a function $f\colon (0,1) \to \mathbb{R}$ such that $f(x) < x$ for all $x \in (0,1)$. Show that $\sup\{f(x)|x \in (0,1)\} \leq 1$. Is it true that $\sup\{f(x) \mid x \in (0,1)\} < 1$? (Prove or give a counterexample.)