MAT 319: HOMEWORK 9

DUE TUESDAY, MARCH 27

1. Show that the following sequences are divergent.
(a) $x_n = \sin(\pi n/4)$
(b) $y_n = (-1)^n + \cos(\pi n/6)$

2. Suppose the sequence (x_n) is unbounded.
(a) Show that there exists a subsequence (x_{n_k}) of (x_n) such that $|x_{n_k}| > k$ for all $k \in \mathbb{N}$.
(b) Show that there exists a subsequence (x_{n_k}) such that $(1/x_{n_k})$ converges to 0.

3. Let (x_n) be a bounded sequence, and let $s = \sup\{x_n : n \in \mathbb{N}\}$. Also, suppose that $s \neq x_n$ for any n. Show that there is a subsequence of (x_n) which converges to s. What if $x_n = s$ for some n?

4. (a) Suppose that $x_n \geq 0$ for all $n \in \mathbb{N}$, and that $\lim x_n = 2$. Find a subsequence of $((-1)^n x_n)$ that converges to 2 and another that converges to -2. Does the sequence $((-1)^n x_n)$ converge?
(b) Suppose that $x_n \geq 0$ for all $n \in \mathbb{N}$, and $((-1)^n x_n)$ is a convergent sequence. Show that the sequence (x_n) is convergent. What's its limit?

5. Give an example of a sequence (x_n) which has subsequences (y_n), (z_n), and (w_n), such that

$$\lim_{n\to\infty} y_n = -\infty, \quad \lim_{n\to\infty} z_n = 0, \text{ and } \quad \lim_{n\to\infty} w_n = 3.$$

Can the sequence (x_n) be convergent?

6. Suppose $\lim x_n/n = 5$. Show that $\lim x_n = +\infty$. (Give a careful proof, arguing from definitions).

7. Give examples of two sequences (x_n) and (y_n) with

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty} y_n = +\infty$$

(and $y_n \neq 0$ for all n), such that
(a) $\lim_{n\to\infty}(x_n/y_n) = +\infty$;
(b) $\lim_{n\to\infty}(x_n/y_n) = 0$;
(c) $\lim_{n\to\infty}(x_n/y_n) = 2$;
(d) the sequence (x_n/y_n) is bounded but divergent.