MAT 319: HOMEWORK 8

DUE TUESDAY, MARCH 20

1. For each of the following sequences, find out whether it is convergent. (Prove your answer.) If convergent, find the limit. (You may use the limit theorems, but you have to explain exactly how you apply them.)

(a) $a_n = \frac{(-1)^n \cos n}{n^2}$.

(b) $b_n = \sqrt{n+1} - \sqrt{n}$.

(c) $c_n = (-1)^n n^2$.

(d) $d_n = (1 + \frac{1}{n})^{2n+1}$.

2. Show that $\lim \frac{n^2}{n!} = 0$.

3. Suppose that (x_n) is a convergent sequence, and the sequence (y_n) is such that for every $\varepsilon > 0$ there exists $K \in \mathbb{N}$ such that $|x_n - y_n| < \varepsilon$ for all $n \geq K$. Does it follow that (y_n) converges?

4. Give a direct proof of the Monotone Convergence Theorem for decreasing sequences that are bounded below. (In Bartle–Sherbert p. 70 this is reduced to the case of increasing sequence by considering $(x_n) = (-y_n)$. Don't do that; rather, give a proof which is similar to the one on p. 69 in part (a)).

5. Let $x_1 = 2$ and $x_{n+1} = 2 - 1/x_n$ for $n \in \mathbb{N}$. Show that (x_n) is bounded and monotone. Find the limit.

6. Let the sequence a_n be defined by $a_1 = 1$, $a_{n+1} = \sqrt{1 + a_n}$.

(a) Show that for any n we have $a_n < a_{n+1} < \Phi$, where Φ is the root of the equation $x = \sqrt{1+x}$.

(b) Show that a_n converges and find the limit.