MAT 319: HOMEWORK 7

DUE TUESDAY, MARCH 13

1. Use the definition of limit to prove that:

(a) $\lim_{n\to\infty} \dfrac{n}{2n-1} = \dfrac{1}{2}$

(b) $\lim_{n\to\infty} \dfrac{\sqrt{n}}{n+1} = 0.$

2. (a) Prove that $\lim x_n = 0$ if and only $\lim |x_n| = 0$.

(b) Give an example to show that in general, (x_n) does not have to converge if $|x_n|$ converges.

3. (a) Give an example of two divergent sequences (x_n) and (y_n) such that their sum $(x_n + y_n)$ converges.

(b) Can you give an example where (x_n) converges, (y_n) diverges, and $(x_n + y_n)$ converges?

(c) Give an example of two divergent sequences (x_n) and (y_n) such that their product $(x_n \cdot y_n)$ converges.

(d) Can you give an example where (x_n) converges, (y_n) diverges, and $(x_n \cdot y_n)$ converges?

4. For each of the following sequences, find out whether it is convergent. If convergent, find the limit. (You may use the limit theorems, but you have to explain exactly how you apply them.)

(a) $a_n = (-1)^n + \frac{1}{n}$.

(b) $b_n = \frac{4n^2+2n-1}{2n^2-3}$

5. Let $I_n = [a_n, b_n]$ be a sequence of nested intervals such that $\lim(b_n - a_n) = 0$. Show that then $\lim a_n = \lim b_n$, and if we denote this common limit by a, then $\bigcap_{n=1}^{\infty} I_n = \{a\}$.