

MAT 319: HOMEWORK 6
DUE TUESDAY, MARCH 6

1. We say that a sequence (x_n) converges to A *very well* if there exists $k \in \mathbb{N}$ such that for any $\varepsilon > 0$, $|x_n - A| < \varepsilon$ for all $n \geq k$. (That is, the number k in the definition of limit does not depend on ε .) Describe all sequences that converge very well to $A = 1$ and prove your answer.
2. Let $x_n = (-1)^n \frac{n+1}{n^4+2}$ for $n = 1, 2, 3, \dots$
 - (a) Find $k \in \mathbb{N}$ such that $|x_n| < \frac{1}{100}$ for all $n \geq k$. (You don't have to find the best possible k .)
 - (b) Prove that the sequence (x_n) converges to 0.
3. Consider the sequence (x_n) , where $x_n = \begin{cases} 1, & n = 1, 4, 7, \dots \\ 2, & n = 2, 5, 8, \dots \\ 3, & n = 3, 6, 9, \dots \end{cases}$.
 - (a) Prove that (x_n) does not converge to 2.
 - (b) Prove that (x_n) does not converge to any $A \in \mathbb{R}$.