MAT 319: HOMEWORK 5

DUE TUESDAY, FEBRUARY 27

1. Let $S = \{\frac{1}{n} - \frac{1}{m} \mid n, m \in \mathbb{N}\}$. Find $\inf S$, $\sup S$ (if they exist). Explain your answer.

2. Let functions $f, g : [0,1] \to \mathbb{R}$ be both bounded. Show that

$$\sup_{x\in[0,1]} (f(x) + g(x)) \leq \sup_{x\in[0,1]} f(x) + \sup_{x\in[0,1]} g(x).$$

Give an example where $\sup_{x\in[0,1]}(f(x) + g(x))$ is not equal to $\sup_{x\in[0,1]} f(x) + \sup_{x\in[0,1]} g(x)$ (for some functions f, g).

What would be the corresponding inequality for infimums?

3. As usual, for a real number a we denote $V_\varepsilon(a) = \{x \mid |a-x| < \varepsilon\}$. Find

$$\bigcap_{n\in\mathbb{N}} V_{1/n}(a).$$

(Prove your answer using the Archimedean Property.)

4. Let $I_n = (-n, +\infty)$, $J_n = (-\infty, -n)$ for $n \in \mathbb{N}$. Find $\bigcap_{n\in\mathbb{N}} I_n$ and $\bigcap_{n\in\mathbb{N}} J_n$.

5. Find the binary representation of the following numbers:

(a) 2/3

(b) 0.625

Explain how this relates to nested intervals.