MAT 319: HOMEWORK 3

DUE TUESDAY, FEB. 13

1. (a) List all the subsets of $S = \{1, 2\}$ (do not forget $\varnothing$ and S itself, both of which are subsets of S).
(b) List all the subsets of $S = \{1, 2, 3\}$. (Try to see a relation with (a) — this should give you an idea for the inductive step below.)
(c) Prove that for all $n \geq 1$, if a finite set S has n elements then it has 2^n different subsets. [**Hint:** Use induction. You should have checked this above for $n = 2$, 3, but you need to do the inductive step.]

2. Show that the set $\mathbb{Z}_{odd}$ of odd (positive and negative) integers is denumerable by
(a) enumerating them and
(b) giving an explicit formula for the corresponding bijection $f \colon \mathbb{Z}_{odd} \to \mathbb{N}$.

3. (a) In class, we proved that the product of two denumerable sets is denumerable. It follows that the set $\mathbb{Z} \times \mathbb{N}$ is denumerable. Give a different proof by explaining how to enumerate the elements of $\mathbb{Z} \times \mathbb{N}$ directly. (You should provide a clear explanation, but you don't have to give an explicit formula for the bijection between $\mathbb{Z} \times \mathbb{N}$ and $\mathbb{N}$.)
(b) Do the same for $\mathbb{Z} \times \mathbb{Z}$.

4. Let S be set of all pairs (q, n), where q is a positive rational and n is an even integer. Prove that S is denumerable. (You can use any theorems we learned in class.)

5. Let A be a set of disks (possibly of different radius) drawn on the standard (x, y) plane, so that any two disks have an empty intersection. (As usual, a disk of radius $r > 0$ centered at a point (x_0, y_0) is the set $\{(x, y) : (x - x_0)^2 + (y - y_0)^2 \leq r^2\}$.) Show that the set A is countable. [**Hint:** every disk contains a point (p, q), where p and q are rational numbers. (Why?)]

6. Explain how to construct a bijection between the open interval $(0, 1)$ and the closed interval $[0, 1]$, showing that the two intervals have the same cardinality (i.e. "the same number of points"). [**Hint:** your bijection should be quite complicated. In particular, don't try to construct a *continuous* bijection. Indeed, we will see later in the course that a continuous bijection between the two intervals does not exist.]