MAT 319: HOMEWORK 12

DUE THURSDAY, MAY 3

1. Suppose functions $f(x)$, $g(x)$ are defined in a neighborhood of c (except perhaps for $x = c$), and $\lim_{x\to c} f = +\infty$ and $\lim_{x\to c} g = -\infty$. Give examples where
 (a) $\lim_{x\to c}(f+g) = -3$;
 (b) $\lim_{x\to c}(f+g) = +\infty$;
 (c) $\lim_{x\to c}(f+g) = -\infty$;
 (d) the function $(f+g)$ is bounded in a neighborhood of c, but $\lim_{x\to c}(f+g)$ does not exist.

2. Suppose $\lim_{x\to+\infty} f(x) = 0$, and the function $g(x)$ is bounded on $\mathbb{R}$. Arguing from definitions, show that $\lim_{x\to\infty} fg = 0$.

3. Let $f(x) = \sqrt{(}x) + 3$.
 (a) Find $\alpha \in \mathbb{R}$ such that $f(x) > 100$ for all $x > \alpha$. (You don't have to find the best possible α, but you have to show that your α works.)
 (b) Show that $\lim_{x\to+\infty} f(x) = +\infty$

4. (a) The function $f(x) = (x-1)(x-2)(x-3)(x-4)(x-5)$ has five roots in the interval $[0,7]$. If the Bisection method is applied on this interval, which of the roots is located?
 (b) Same question for $g(x) = (x-2)(x-3)(x-4)(x-5)(x-6)$ on the interval $[0,7]$.

5. Suppose that the function $f : \mathbb{R} \to \mathbb{R}$ is continuous on $\mathbb{R}$ and that $\lim_{x\to-\infty} f(x) = 0$ and $\lim_{x\to+\infty} f(x) = 0$. Prove that f is bounded on $\mathbb{R}$ and attains either a maximum or minimum on $\mathbb{R}$. Give an example to show that both a maximum and a minimum need not be attained.

6. Let $f, g : [0,1] \to \mathbb{R}$ be two continuous functions. Suppose that for a sequence (x_n) of real numbers $x_n \in [0,1]$ we have

$$f(x_n) = g(x_n) + \frac{1}{n}.$$

Prove that $f(y) = g(y)$ for some $y \in [0,1]$.
Hint: Use Bolzano–Weierstrass theorem to choose a convere-gent subsequence out of (x_n).