MAT 319 Introduction to Analysis

Homework 7

due Thursday, March 22

Please prove (or explain as appropriate) all your answers.

Question 1. Prove that the function $f(x) = 4x - 5$ is continuous at every point x_0,
(a) using the sequences definition.
(b) using the ϵ-δ definition.

Question 2. In class, we considered the function $f(x)$, where

$$f(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0. \end{cases}$$

Prove that this function is continuous at $x_0 = -\frac{1}{2}$.
(a) using the sequences definition.
(b) using the ϵ-δ definition.

Question 3. Consider the function $g(x)$, where

$$g(x) = \begin{cases} 1 & \text{if } x \text{ is a rational number} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$

Prove that $g(x)$ is discontinuous at every point,
(a) using the sequences definition.
(b) using the ϵ-δ definition.

Question 4. Consider the function $h(x)$, where

$$g(x) = \begin{cases} x^2 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0. \end{cases}$$

Is $h(x)$ continuous at 0? Prove your answer (from any definition).

Question 5. Suppose that $f(x)$ is a continuous function. Prove (from any definition) that the function $7f(x)$ is also continuous.