

MAT 319 Introduction to Analysis

**Homework 5**

due Thursday, March 1

Please prove (or explain as appropriate) all your answers.

**Question 1.** In class, we showed by induction that  $2^n > n$  for every positive integer  $n$ . Using this, show that

- (a) the sequence  $(2^n)$  diverges to  $+\infty$
- (b) the sequence  $(\frac{1}{2^n})$  converges to 0.

**Question 2.** Prove that  $\sqrt{5}$  is irrational. (Do not refer to Theorem 2.2 from the book that we didn't cover. Give a proof from scratch.)

**Question 3.** Let  $(x_n)$  be an increasing sequence.

- (a) Prove that  $(x_n)$  is bounded below.
- (b) Suppose that  $(x_n)$  is not bounded above. Prove that  $x_n$  diverges to  $+\infty$ .

**Question 4.** (a) Prove that  $1 \neq 0$ .

Use the algebraic axioms A1-A4, M1-M4, DL on p. 13 as well as parts (i) and (ii) in Theorem 3.1. You can assume that the set  $\mathbb{R}$  of real numbers has more than one element. Please give a proof from scratch, justify every step, and do not use any of the statements not listed above.

(b) Prove that  $0 \leq 1$ .

Use only the axioms A1-A4, M1-M4, DL, O1-O5 on p.13, and (i) and (ii) in Theorem 3.1 if needed. Justify every step.

Please also do questions 4.6 and 4.7(a) from the textbook.