MAT 319 Introduction to Analysis

Homework 3

due Thursday, February 16

Please prove (or explain as appropriate) all your answers.

Question 1. Find the following limits using limit laws (section 9). Explain carefully which theorems you used at each step.

(a)

$$x_n = \frac{2n-1}{3n+2}$$

(b)

$$x_n = \frac{7n^3 - n^2 + 1}{2n + 5n^3 - 3}$$

(c)

$$x_n = \frac{n}{n^4 + n^3 + n^2 - n + 1}$$

Question 2. (a) Suppose that (x_n) converges and (y_n) does not. Prove that $(x_n + y_n)$ diverges.

(b) Suppose that both sequences (x_n) and (y_n) diverge. Is it possible that $(x_n + y_n)$ converges? (Prove or give a counterexample.)

Question 3. Suppose that the sequence (x_n) converges to 4. Prove carefully that $(\sqrt{x_n})$ converges to 2. (Check also that for sufficiently large n, $\sqrt{x_n}$ is defined, i.e. x_n is non-negative.)

For your proof, the identity $(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) = a - b$ will be helpful.

Question 4. (a) Suppose that the sequence (x_n) converges to A, the sequence (y_n) converges to B, and $x_n \leq y_n$ for every n. Prove that $A \leq B$.

(b) Suppose that, as in part (a), $x_n \to A$, $y_n \to B$, but now $x_n < y_n$ for every n. (The inequality is strict.) Give an example showing that the equality $A = B$ is possible.