MAT 319 Introduction to Analysis

Homework 7

due Thursday, April 26

Please prove (or explain as appropriate) all your answers.

Question 1. (a) Prove that a constant function is differentiable at any point. Find its derivative.

(b) Suppose that $f(x)$ is differentiable at a. **Arguing from definitions,** prove that the function $g(x) = 3f(x) + 2$ is differentiable at a, and find $g'(a)$.

Question 2. (a) Suppose that $g(x)$ is differentiable at a, and $g(a) \neq 0$. **Arguing from definitions,** prove that the function $h(x) = \frac{1}{g(x)}$ is differentiable at a, and find its derivative.

(b) Using part (a) and the product rule, prove quotient rule: if f, g are differentiable at a, then $\frac{f}{g}$ is also differentiable at a, and

$$\left(\frac{f}{g}\right)'(a) = \frac{f'(a)g(a) - f(a)g'(a)}{(g(a))^2}.$$

Please use the proposed strategy. Do not repeat the proof from the textbook.

Question 3. (a) Using the product rule and induction, show that

$$(x^n)' = nx^{n-1} \text{ for all natural } n.$$

If you are not familiar with induction, just prove the formula for $n = 3, 4, 5$. (**The textbook has a different proof. Do not repeat it.**)

(b) Using Question 2, find (with proof) $(x^{-5})'$.

Please also do questions 28.7 and 28.8 from the textbook.