MAT 211 Linear Algebra

Final Exam Topics

This is the description of problems that may appear on the final exam. (The actual practice questions can be found in the textbook or past homework or exams.) The goal of this list is to help you to focus on essential skills and to streamline your exam preparation. The list below is quite large; not all questions below will be included, and those that will be included may contain fewer parts. Topics not listed below will not be included in the test.

1. For a system of linear equations:
write it in a matrix form
find rref
determine the rank, the number of free variables
find all solutions of the system
describe the collection of solutions geometrically (a line, plane, etc)

2. Determine whether a given matrix is invertible
(by computing the determinant or working with rref)
find the inverse matrix
use the inverse matrix to solve linear systems

3. Given two matrices
multiply them if possible
interpret multiplication as composition of linear transformations

4. For a collection of vectors (in $\mathbb{R}^n$ or abstract vector space),
determine whether they are linearly independent
whether they form a basis
find the dimension of their span
find coordinates of a vector in a given basis

5. For an abstract vector space,
determine whether a given collection of vectors forms a subspace
determine whether a given transformation is linear
find the dimension of the space (or of given subspace)

6. For a linear transformation of $\mathbb{R}^n$ given by a matrix in a standard basis,
find the matrix in some other given basis.
For a linear transformation given in non-standard basis of $\mathbb{R}^n$,
find matrix in the standard basis.
Find the change of basis matrix. (In both cases.)

7. For a linear transformation of $\mathbb{R}^n$ described geometrically (eg reflection etc),
find its matrix (in a standard or some other given basis).
For a linear transformation in a vector space (eg space of matrices or polynomials),
find its matrix in a given basis.

8. For a given linear transformation in $\mathbb{R}^n$:
find dimensions of kernel and image
find a basis for ker, im
verify rank-nullity theorem

9. Geometry of vectors:
find the length of a given vector
find dot product of two vectors
find angle between two vectors
determine whether two vectors are orthogonal
verify the Pythagorean theorem for orthog. vectors
find area/volume of parallelogram/parallelepiped formed by given vectors
find orthogonal complement to a given subspace

10. Given a basis in $\mathbb{R}^n$,
apply the Gram-Schmidt method to obtain an orthonormal basis

11. For a given matrix
find the characteric polynomial and eigenvalues
find eigenvectors
determine whether the matrix is diagonalizable
find the change of basis matrix to pass from the original matrix to the diagonal one