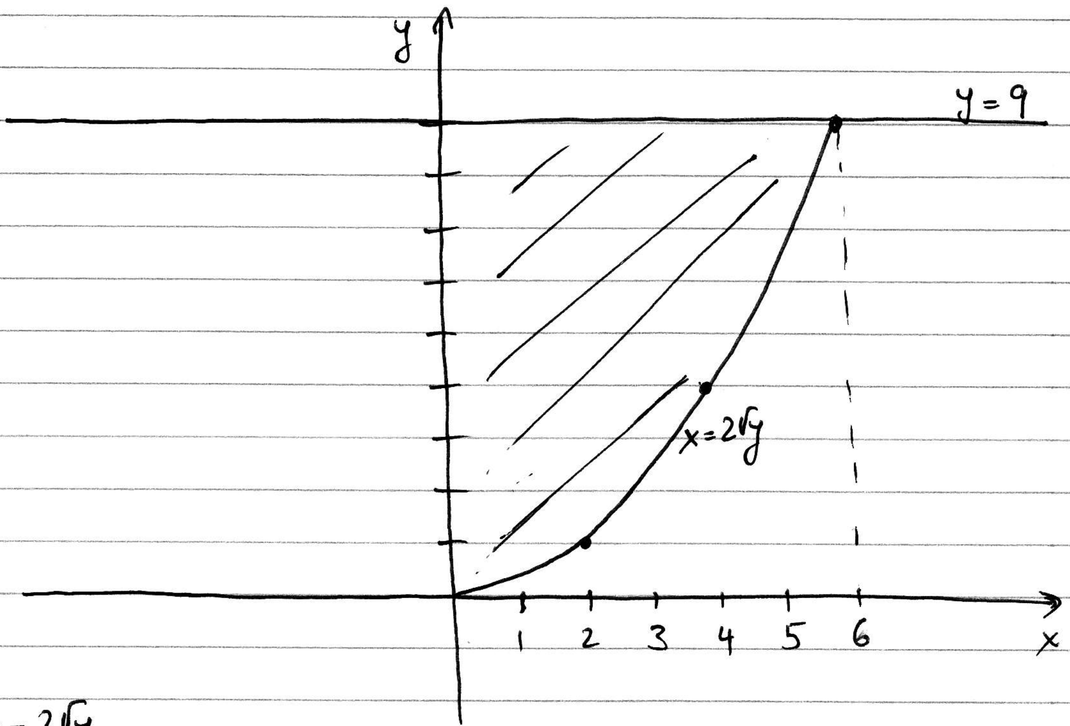
Volume Problems

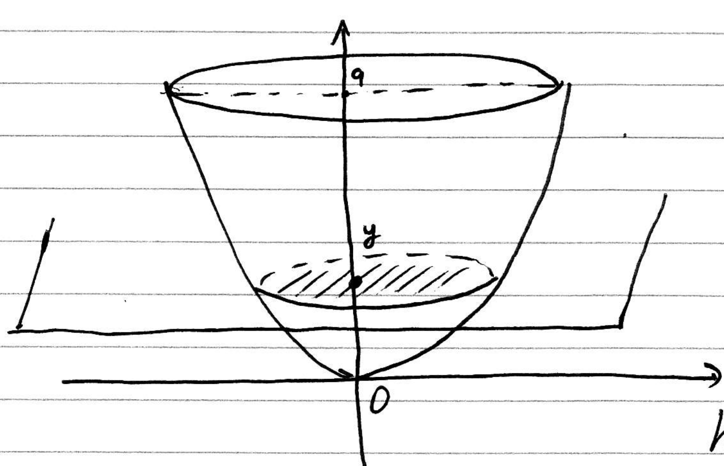
6.2 #3 $x = 2\sqrt{y}$ $x=0$ $y=9$ around y-axis



$x = 2\sqrt{y}$

$(y=1 ; x=2)$ → $(2;1)$ $(y=4 \quad x=4)$ → $(4;4)$ $y=9 \quad x=6$ → $(6;9)$

$x=0$ → y-axis ⇒ The region is shaded



The body looks like ← like

We have to slice horizontally.

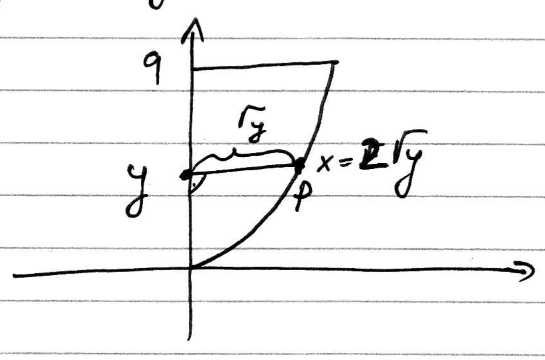
The slices are disks.

$$V = \int_0^9 A(y)\,dy$$

An arbitrary (slice) disk through y will be a circle with radius r_y

$\Rightarrow A(y) = \pi \cdot r_y^2$

To find $r_y \rightarrow$ from the sketch of the region



$r_y = x_P$ (r_y is the x-coordinate of the point P, which is on $x = 2\sqrt{y}$)

$\Rightarrow r_y = 2\sqrt{y} \quad \Rightarrow r_y^2 = 4y$

$\Rightarrow A(y) = \pi \cdot 4y$

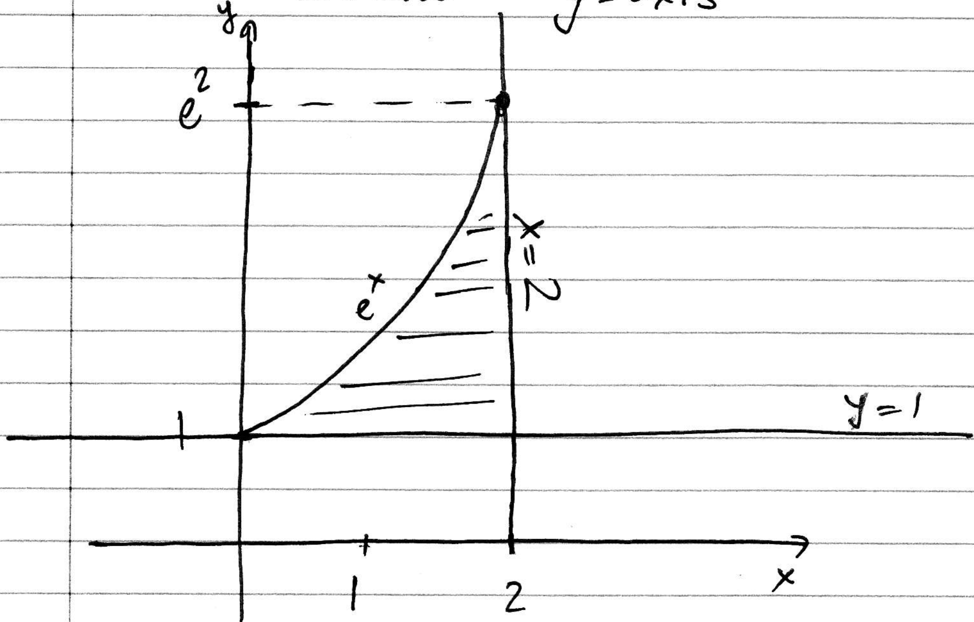
$$\Rightarrow V = \int_0^9 \pi \cdot 4y\,dy = 4\pi \int_0^9 y\,dy = 4\pi \cdot \frac{y^2}{2}\Big|_0^9$$

$$= 162\pi.$$

Example 2:

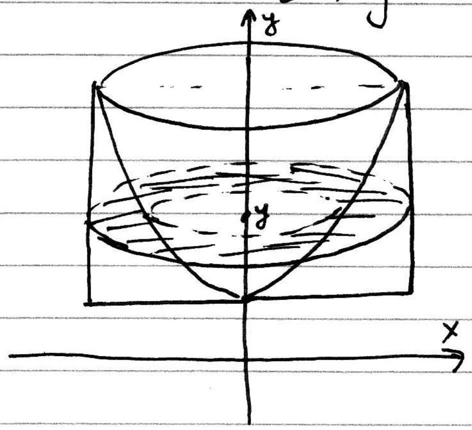
$y = e^x \; ; \; y = 1 \; ; \; x = 2$

around y-axis



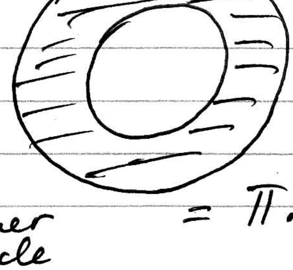
$y = e^x$ $\quad x = 1 \to y = e$

$\quad x = 2 \to y = e^2 \approx 7.3$



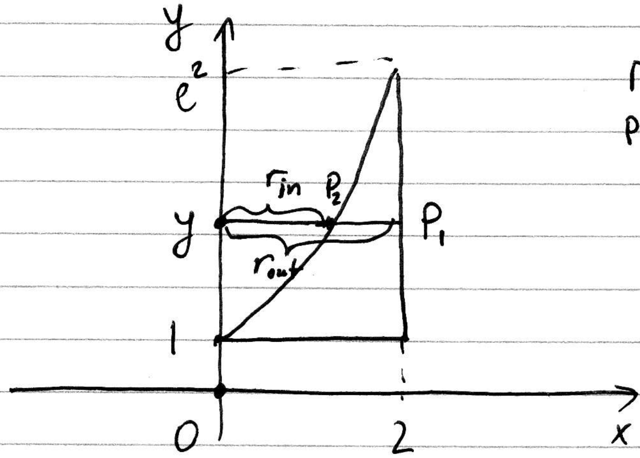
The solid looks ← roughly like this

The slices are horizontal ; They are washers :



$A(y) = A_{slice} =$

$A_{slice} = A_{\substack{outer \\ circle}} - A_{\substack{inner \\ circle}} = \pi \cdot r_{out}^2 - \pi \cdot r_{in}^2$



r_{out} = distance between point P_1 and y-axis

$r_{out} = x_{P_1}$

(the x-coordinate of the point P_1)

P_1 is on the line $x=2$

$\Rightarrow r_{out} = 2$

$r_{in} = x_{P_2}$ (the x-coord. of point P_2)

P_2 is on $y=e^x$ $\Rightarrow r_{in} = x$, but we need it in terms of y

$\Rightarrow$ from $y=e^x \rightarrow x = \ln y$

$\Rightarrow r_{in} = \ln y$

$\Rightarrow A_{slice} = \pi \cdot 2^2 - \pi \cdot (\ln y)^2 = 4\pi - \pi \cdot (\ln y)^2$

$$\Rightarrow V = \int_1^{e^2} A(y)\,dy = \int_1^{e^2} \left(4\pi - \pi \cdot (\ln y)^2\right) dy$$

$$= \int_1^{e^2} 4\pi\,dy - \pi \int_1^{e^2} (\ln y)^2\,dy = \text{------}$$

(Have to finish the integral

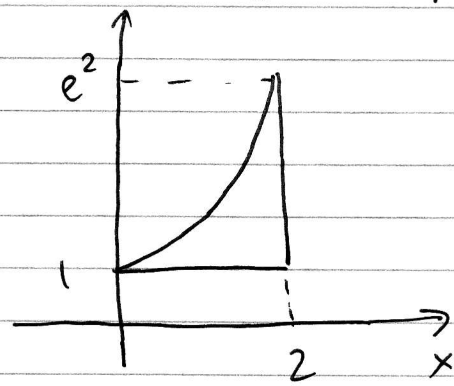
$\int (\ln y)^2\,dy$ (which is 2 times by parts)

Nothing so hard like this integral is on the quiz.

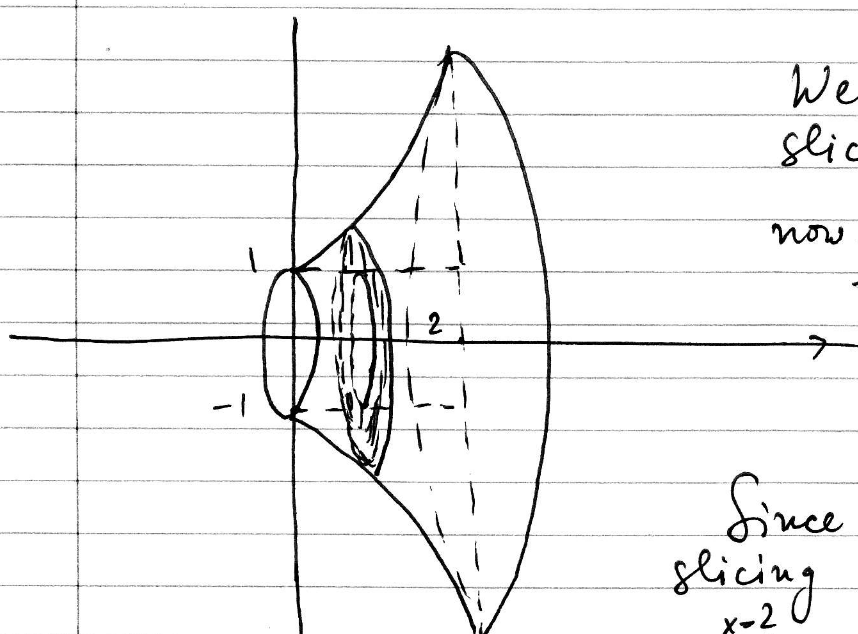
Example 3: $y = e^x$; $y = 1$; $x = 2$

around the <u>x-axis</u>

It is the same region

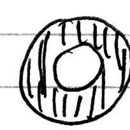


But the solid body is:
(rotating around x-axis)



We have to slice <u>vertically</u> now.

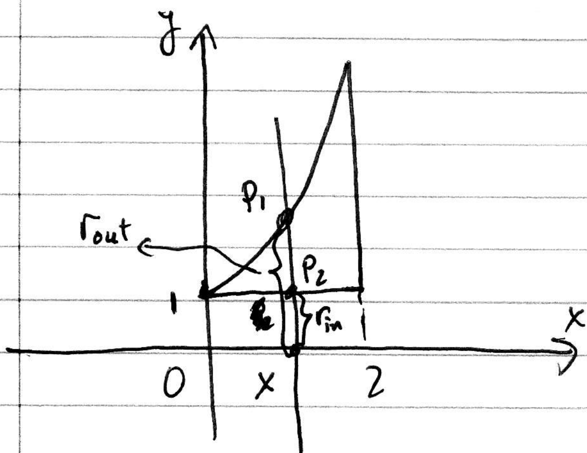
The slices are washers.



Since we are slicing vertically

$$V = \int_{x=0}^{x=2} A(x)\,dx$$

$$A(x) = A_{slice} = A_{\substack{outer\\circle}} - A_{\substack{inner\\circle}} = \pi \cdot r_{out}^2 - \pi \cdot r_{in}^2$$



r_{in} = distance between P_2 & x-ax.

$r_{in} = y_{P_2}$

(the y-coordinate of the point P_2)

P_2 is on the line $y=1$

$\Rightarrow r_{in} = 1$

$r_{out} = y_{P_1}$; the point P_1 is on $y = e^x$

$\Rightarrow r_{out} = e^x$

$$\Rightarrow A(x) = \pi \cdot (e^x)^2 - \pi \cdot 1^2 = \pi \cdot e^{2x} - \pi$$

$$V = \int_0^2 (\pi \cdot e^{2x} - \pi)\, dx = \pi \int_0^2 e^{2x}\, dx - \pi \int_0^2 1\, dx$$

$$= \pi \cdot \frac{e^{2x}}{2} \Bigg|_{x=0}^{2} - \pi \cdot x \Big|_0^2 = \pi \cdot \frac{(e^4 - 1)}{2} - 2\pi$$

$$V = \frac{\pi \cdot e^4}{2} - \frac{5\pi}{2}$$