Unrealized opportunities.
Topological, quantum and categorification invariants in 4D

Oleg Viro

April 1, 2011

Playing topology
● Similarity between Seiberg-Witten and Alexander
● Looking for a space
● Vassiliev invariants for 4-manifolds?
Playing Quantum Topology
Playing Categorification
Real problem

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$\operatorname{codim}_{\Omega X} KX = 2$, hence $\operatorname{in}_* : \pi_1(KX) \to \pi_1(\Omega X)$ is onto.

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A loop in X with the homotopy class of a spanning disk.

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Playing Quantum Topology

- TQFT
- Unused opportunities

Playing Categorification

Real problem

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● Categorify state sums

● 2-links

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unless the initial knot was trivial.

Khovanov homology and other link homology
give rise to invariants of the links with self-intersections.

This requires extension of the Khovanov homology
to cobordisms with transverse self-intersections.

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Real problem

The real problem

How can we make American teachers familiar with
the mathematics that they are supposed to teach?

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